Christ Church Grammar School

2021
TEST 5

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

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Instructions to candidates

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2. Write your answers in this Question/Answer Booklet using a blue or black pen. Do not use erasable or gel pens.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(5 marks)**

(a) To investigate people's attitudes to control of gun ownership, a TV station conducts a phone-in poll, where people are asked to telephone one number if they are in favour of tighter gun control, and another if they are against. Is this an appropriate method of choosing a random sample? Give reasons for your answer. (2 marks)

(b) In a certain school, 35% of the students travel on the school bus. A group of 100 students were selected in a random sample, and 42 of them travel on the school bus. In this context,

(i) describe the population. (1 mark)

(ii) determine the value of the population proportion p. (1 mark)

(iii) determine the value of the sample proportion $\hat{p}$. (1 mark)

See next page

Question 2 **(6 marks)**

(a) Determine the anti-derivative of $(2x + \frac{1}{x})(2x - \frac{1}{x})$ (3 marks)

(b) $y = x + 1$ is a tangent to the curve $y = ax + b\sin(x)$ at the point $\left(\frac{\pi}{2}, 1 + \frac{\pi}{2}\right)$.

Determine the values of a and b. (3 marks)

Question 3 **(4 marks)**

Sketch, on the axes below, the following function with ***all*** of the properties listed.

$y = f(x)$ such that:

- $f(-1) = 0$
- $f'(0) = 0$
- $f(x) > 0 \; for \; x > -1$
- $f(x) < 0 \; for \; x < -1$
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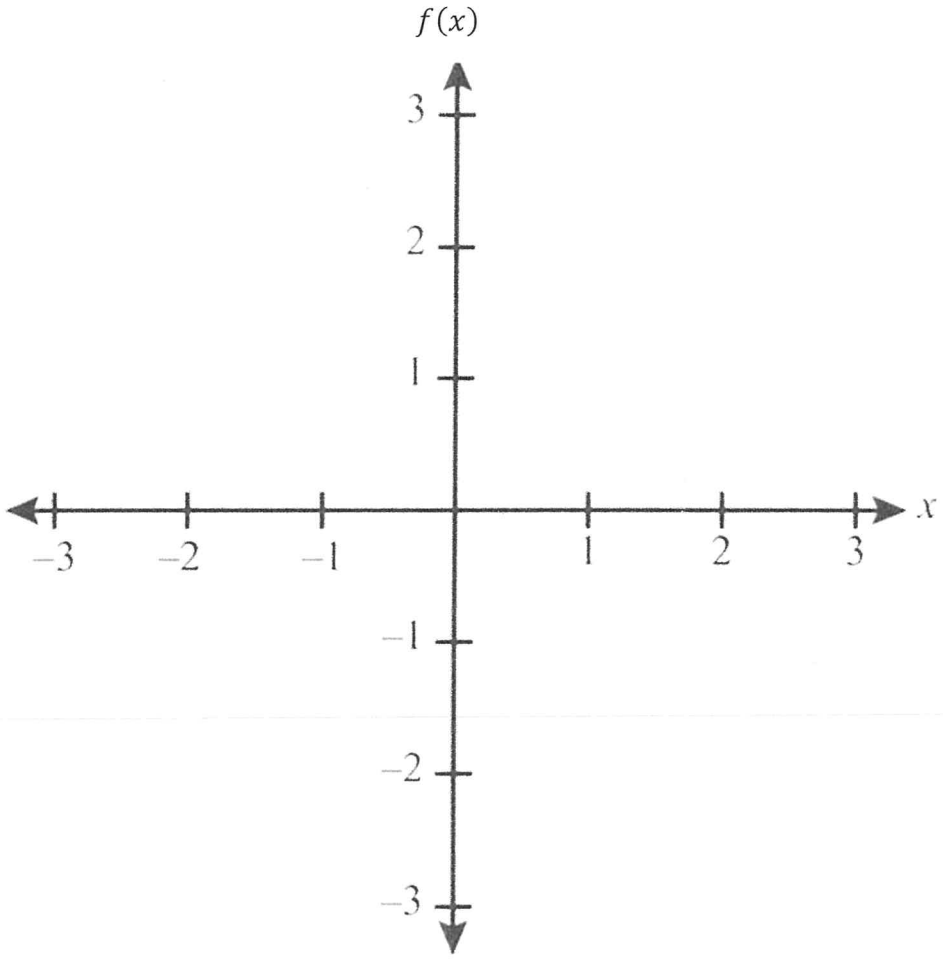

End of questions

Additional working space

Question number: _____

Christ Church Grammar School

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TEST 5

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

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See next page

Question 4 **(7 marks)**

To predict the number of people in Australia having a certain disease, a random sample of 1600 people was tested, in which 56 were found to have the disease.

(a) Determine the sample proportion, $\hat{p}$, of people having the disease. (1 mark)

(b) Determine the 90% confidence interval for the proportion of the population that have the disease. (1 mark)

(c) Another random sample is taken. A 90% confidence interval is wanted with a margin of error of at most 0.5%. Determine the minimum sample size required. (3 marks)

(d) A survey in an isolated town, with a population of 12030, finds that 2503 people have a disease. Compare these findings with that of the population. (2 marks)

See next page

Question 5 **(4 marks)**

Let the proportion of parents in a college that support a four-day school week be π. A random sample of 200 parents was selected and 78 indicated that they support the proposal. Calculate the level of confidence for a confidence interval for π with an error of ± 0.1.

See next page

Question 6 **(6 marks)**

The amount R, in grams, of a radioactive substance X remaining at time t years is given by $R = 100e^{-0.01t}, t \geq 0$.

(a) Determine the initial amount of X. (1 mark)

(b) Determine the continuous rate of decay for X as a percentage. (1 mark)

(c) Determine the amount of X that has decayed after 100 years. (2 marks)

(d) Determine the time it will take for the amount of X to be halved.
(i.e. the half life of X)

(2 marks)

Question 7 **(4 marks)**

Determine the value(s) of k for which $y = e^{kx}$ is a solution of the equation:

$$2 \times \frac{d^2y}{dx^2} - \frac{dy}{dx} - 3y = 0$$

See next page

Question 8 **(3 marks)**

In the domain $0 \leq x \leq \pi$, calculate, correct to 3 decimal places, the coordinates of the position on the curve $y = 3sinx - sin^3x$ where the slope of the curve is $\frac{3}{8}$.

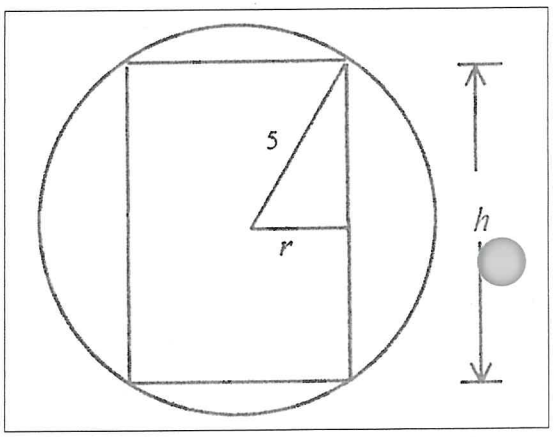
See next page

Question 9 **(6 marks)**

A metal worker is required to cut a circular cylinder from a solid sphere of metal of radius 5 cm. The diagram shows a cross section of the sphere and cylinder.

Let V be the volume of the sphere.

(a) Show that $r = \frac{1}{2}\sqrt{100 - h^2}$ (2 marks)



(b) Calculate the value of h that maximises the volume of the cylinder. Justify the optimisation. (4 marks)

End of questions

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TEST 5

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name - Solutions -

Teacher's name _____

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Question 1 **(5 marks)**

(a) To investigate people's attitudes to control of gun ownership, a TV station conducts a phone-in poll, where people are asked to telephone one number if they are in favour of tighter gun control, and another if they are against. Is this an appropriate method of choosing a random sample? Give reasons for your answer. (2 marks)

No, Self selection bias ✓ [NO]

Only interested parties call in

May call several times ✓ [Reason must incl BIAS]

(b) In a certain school, 35% of the students travel on the school bus. A group of 100 students were selected in a random sample, and 42 of them travel on the school bus. In this example:

(i) What is the population? (1 mark)

All students at the school ✓

(ii) What is the value of the population proportion p ? (1 mark)

0·35 ✓

(iii) What is the value of the sample proportion $\hat{p}$? (1 mark)

0.42 ✓

5

Question 2 **(6 marks)**

(a) Determine the anti-derivative of $(2x + \frac{1}{x})(2x - \frac{1}{x})$ (3 marks)

$\int 4x^2 - x^{-2}\, dx$

$= \frac{4x^3}{3} + x^{-1} + c$

$= \frac{4x^3}{3} + \frac{1}{x} + c$

✓ x^3 term
✓ $\frac{1}{x}$ term
✓ $+c$

(b) $y = x + 1$ is a tangent to the curve $y = ax + b\sin(x)$ at the point $\left(\frac{\pi}{2}, 1 + \frac{\pi}{2}\right)$.

Determine the values of a and b. (3 marks)

$\frac{dy}{dx} = a + b\cos x$ ✓ $\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx} = 1$ when $x = \frac{\pi}{2}$ $\Rightarrow$ $1 = a + b.\cos\frac{\pi}{2}$

$\therefore a = 1$ ✓ (a value)

Also $(\pi/2, 1+\pi/2)$ on curve

$1 + \frac{\pi}{2} = \frac{\pi a}{2} + b\sin(\pi/2)$

$1 + \frac{\pi}{2} = \frac{\pi}{2} + b$

$\therefore b = 1$ ✓ (b-value)

/6

Question 3 **(4 marks)**

Sketch, on the axes below the following function with ***all*** of the properties listed.

$y = f(x)$ such that:

- $f(-1) = 0$
- $f'(0) = 0$
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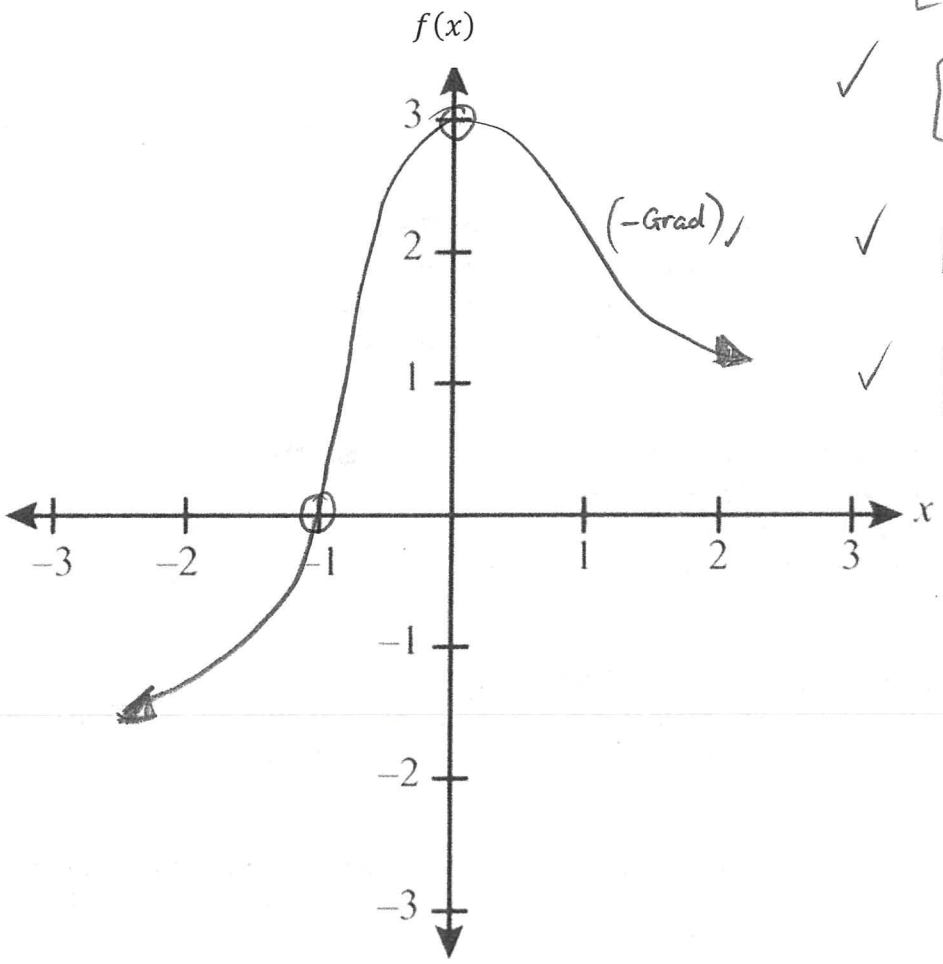

✓ [x-int]

✓ [TP @ x=0]

✓ [f'(x)<0, x>0]

✓ [General shape]

4

End of questions

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Question 4 **(7 marks)**

To predict the number of people in Australia having a certain disease, a random sample of 1600 people was tested, in which 56 were found to have the disease.

(a) Determine the sample proportion, $\hat{p}$, of people having the disease. (1 mark)

$\frac{56}{1600} = 0.035$ ✓

(b) Determine the 90% confidence interval for the proportion of the population that have the disease. (1 marks)

$0.027744 \leq p \leq 0.04256$ ✓

[Clad]

(c) Another random sample is taken. A 90% confidence interval is wanted with a margin of error of at most 0.5%. Determine the minimum sample size required. (3 marks)

90% $\Rightarrow K = 1.645$

$0.5\% = 0.005$ ✓ [ME]

$\therefore 0.005 = 1.645\sqrt{\frac{0.035(1-0.035)}{n}}$ ✓ [EQU]

$n = 3655.84$

$n = 3656$ ✓ [Round correctly]

(d) A survey in an isolated town, with a population of 12030, finds that 2503 people have a disease. Compare these findings with that of the population. (2 marks)

$\frac{2503}{12030} = 0.2081$ ✓ [proportion]

Not consistent. Bias due to town being isolated... etc ✓ [Explanation]

See next page

Question 5 **(4 marks)**

Let the proportion of parents in a college that support a four-day school week be π. A random sample of 200 parents was selected and 78 indicated that they support the proposal. Find the level of confidence for a confidence interval for π with an error of ± 0.1.

$\hat{\pi} = \frac{78}{200} \Rightarrow 0.39$ ✓ [Proportion]

As $n > 30$ by CLT $\hat{\pi} \sim$ Normal

$z_c \times \sqrt{\frac{0.39(1-0.39)}{200}} = 0.1$

$z_c = 2.89946$ ✓ [z-value]

$\therefore P(-2.89946 \leq z \leq 2.89946)$ ✓ [$\pm z_c$]

$= 0.99627$

∴ Level of Confidence is 99.6% ✓ [C.I]

4

Question 6 **(6 marks)**

The amount R, in grams, of a radioactive substance X remaining at time t years is given by $R = 100e^{-0.01t}, t \geq 0$.

(a) Find the initial amount of X. (1 mark)

Initial value $= 100e^{-0.01(0)}$

$= 100\,g$ ✓

(b) Find the continuous rate of decay for X as a percentage. (1 mark)

X is 0.01×100 $= 1\%$ per year ✓

(c) Find the amount of X that has decayed after 100 years. (2 marks)

$t = 100$ amount remaining is $R = 100e^{-0.01(100)}$

$= 36.79\,g$ ✓ [Remain]

$\therefore 100 - 36.79 = 63.2\,g$ decayed ✓ [Decay]

(d) Find how long it will take for the amount of X to be halved. (i.e. the half life of X) (2 marks)

When X is halved $R = 50g$

$100e^{-0.01t} = 50$

$e^{-0.01t} = \frac{1}{2}$ ✓ 69.31471806

$t = 69.3$ yrs ✓

6

Question 7 **(4 marks)**

Find the value(s) of k for which $y = e^{kx}$ is a solution of the equation

$$2 \times \frac{d^2y}{dx^2} - \frac{dy}{dx} - 3y = 0$$

Must show 1st & 2nd derivatives

✓ $\frac{dy}{dx} = ke^{kx}$

✓ $\frac{d^2y}{dx^2} = k^2e^{kx}$

$2k^2e^{kx} - ke^{kx} - 3e^{kx} = 0$ ✓ [sub in]

$e^{kx}(2k^2 - k - 3) = 0$

$e^{kx}(k+1)(2k-3) = 0$

$e^{kx} = 0$ ↓ No Soln ; $k = -1$ OR $k = \frac{3}{2}$ ✓ [Both k-values]

4

Question 8 **(3 marks)**

In the domain $0 \leq x \leq \pi$, find, correct to 3 decimal places, the coordinates of the position on the curve $y = 3sinx - sin^3x$ where the slope of the curve is $\frac{3}{8}$.

$\frac{dy}{dx} = -3\cos x \cdot \sin^2 x + 3\cos x$ ✓ [dy/dx]

$\frac{3}{8} = -3\cos x \cdot \sin^2 x + 3\cos x$

$x = 1.047$ ✓ [x-value] [-1 if no 3dp]

$\therefore y = 1.948$ ✓ [y-value] OR 1.949

if all x decimal places used.

3

Question 9 **(6 marks)**

A metal worker is required to cut a circular cylinder from a solid sphere of metal of radius 5cm. The diagram shows a cross section of the sphere and cylinder.

Let V be the volume of the sphere.

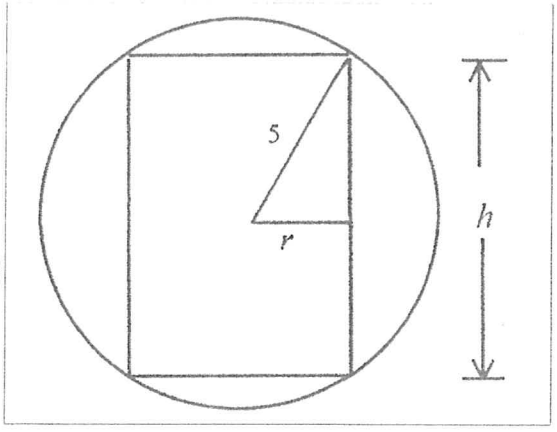
(a) Show that $r = \frac{1}{2}\sqrt{100 - h^2}$ (2 marks)

$5^2 = r^2 + (h/2)^2$ ✓ [Pythag]

$r^2 = 25 - \frac{h^2}{4}$

$r^2 = \frac{100 - h^2}{4}$ ✓ [Re-arranges]

$r = \frac{1}{2}\sqrt{100 - h^2}$



(b) Find the value of h that maximises the volume of the cylinder. Justify the optimisation. (4 marks)

$V = \pi r^2 h$

$V = \pi \left(\frac{100 - h^2}{4}\right) h$ ✓ [Subs r^2]

$\frac{dV}{dh} = \frac{1}{4}\pi(100 - 3h^2)$ ✓ [Shows $\frac{dV}{dh}$]

$\frac{dV}{dh} = 0$

$h = \frac{10}{\sqrt{3}}$ (5.77) ✓ [Finds h value]

Justify

$\frac{d^2V}{dh^2} = \frac{1}{4}\pi(-6h)$

$\left.\frac{d^2V}{dh^2}\right|_{h=\frac{10}{\sqrt{3}}} < 0$ ✓ $\left[\frac{d^2V}{dh^2} < 0\right]$

$\therefore h = \frac{10}{\sqrt{3}}$ is MAX.

6

End of questions